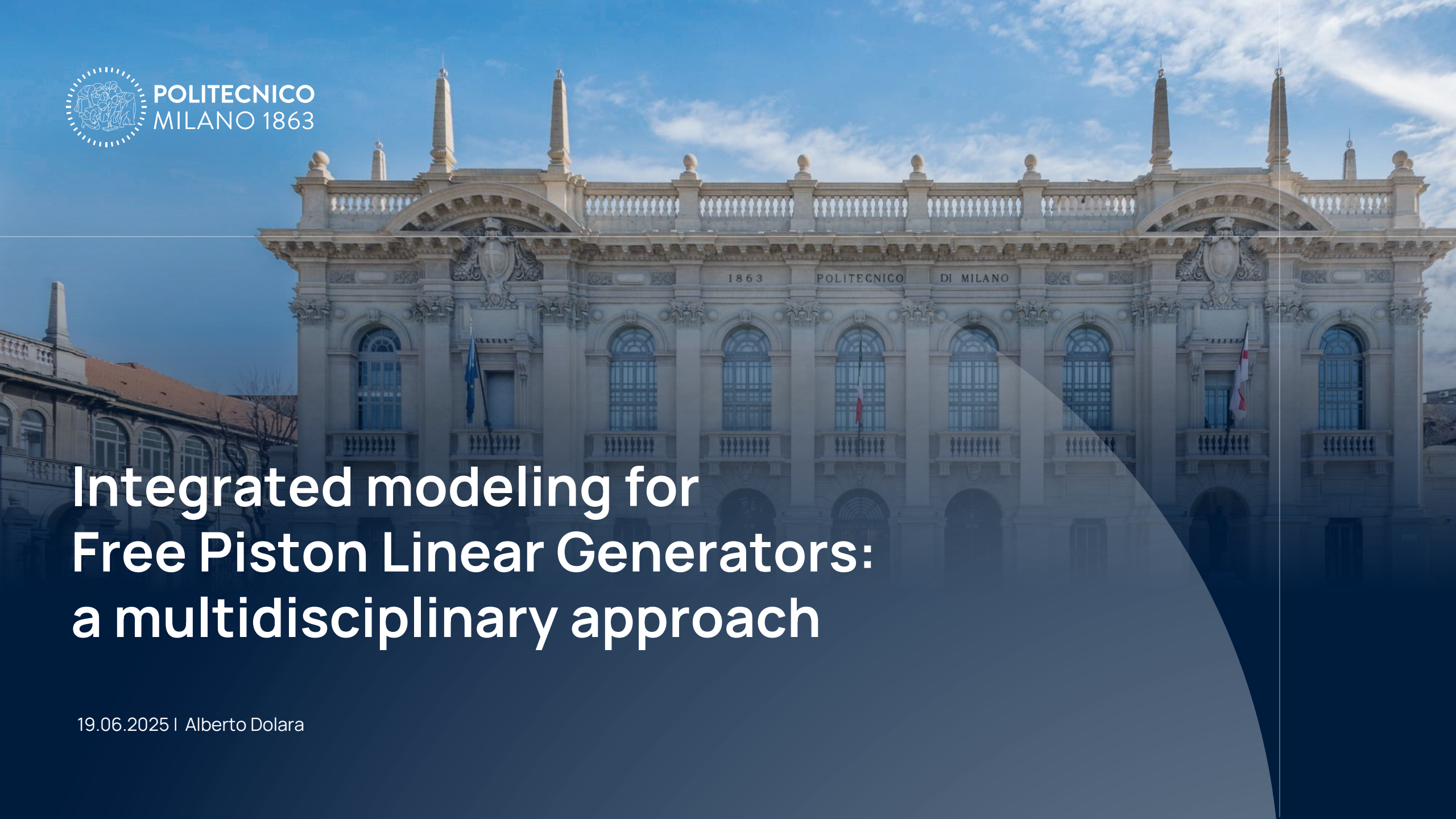


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Integrated modeling for Free Piston Linear Generators: a multidisciplinary approach

19.06.2025 | Alberto Dolara

A **multidisciplinary** approach is necessary to make an **integrated model** of the Free Piston Linear Generator able to properly represent and simulate its complexity.

Academic whim or *real necessity?*

Overview

01

Free-Piston Linear Generator

Question: *Provide a simple definition of a Free Piston Linear Generator.*

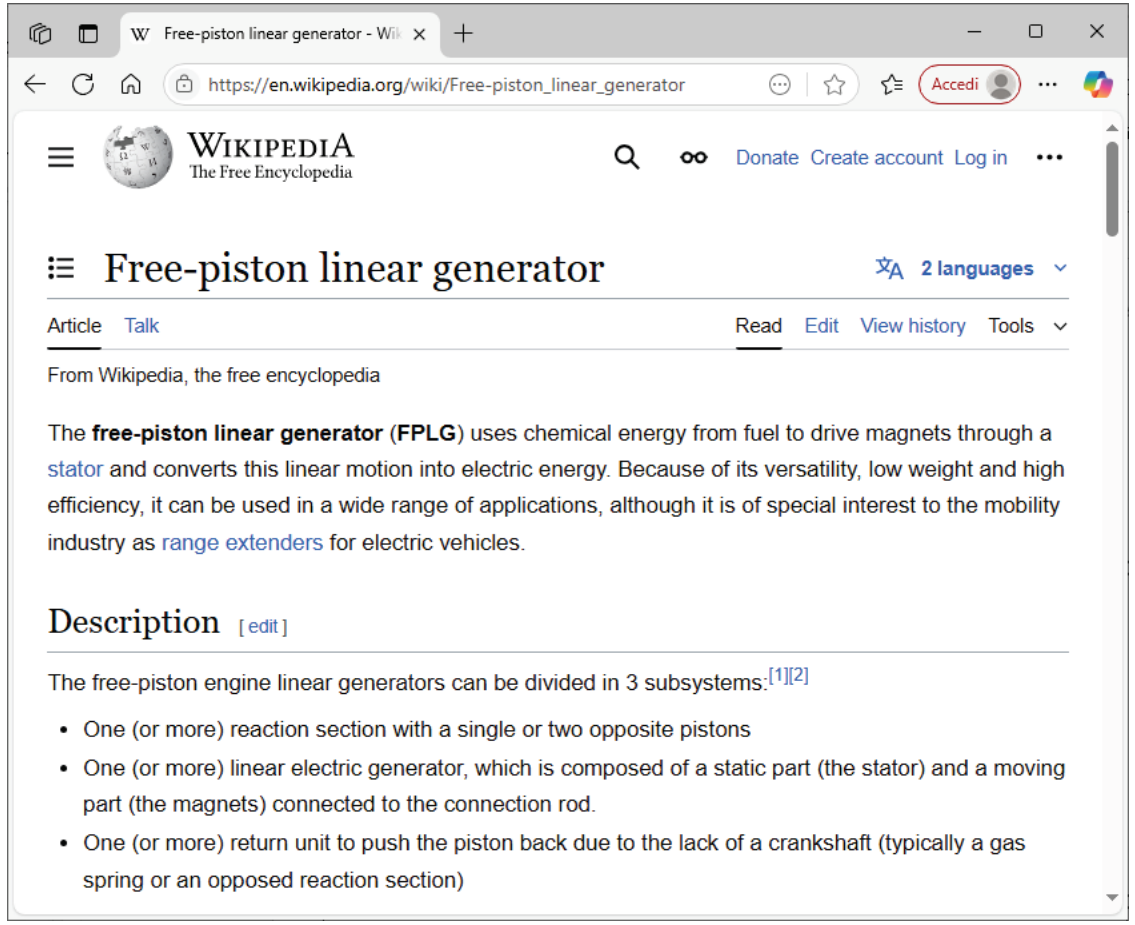
Answer 1

The Free Piston Linear Generator is a **#system** able to **#convert** the chemical energy of a fuel into electrical energy through a **#piston #mechanically coupled** to a **#linear electric machine.** It consists of the following **#subsystems**:

- Internal combustion engine (ICE)
- Linear electric machine (LEM)
- Rebound device (usually a gas spring)

Answer 2

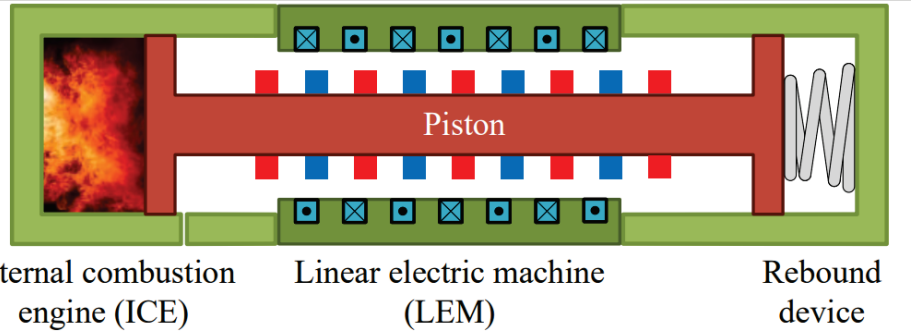
Ask *Google* and *Wikipedia*...



Free Piston Linear Generator modeling

02

Free-Piston Linear Generator



#mechanically coupled

To determine the motion of the translator it is necessary to determine the set of forces acting on it.

Newton's Second Law is the first step towards the FPLG dynamic model:

$$M\frac{d^2x}{dt^2} = F_{ice} - F_{gs} - F_{lem} - F_f$$

Newton's second law recalls for a modeling approach based on the interaction between subsystems:

- F_{ice} is pressure force generated in the ICE;
- F_{gs} is the pressure force exerted by the GS;
- F_{lem} is the electromagnetic force generated by the LEM.
- F_f is the friction force between the piston(s) and the inner cylinder.

These forces depend on the position and on the speed of the translator, and they have to be determined by proper subsystems models.

Free-Piston Linear Generator – ICE model

It is the most studied subsystem in literature, with a lot of modeling approaches characterized by different accuracy and computation burden:

- **0D models**: the cylinder volume is considered as a 0D system where lumped parameters models represents the thermodynamical and chemical processes within the cylinder. Scavenging and combustion processes can be represented with a certain degree of approximation.
- **CFD models**: based on 1D, 2D, or 3D approaches, allows to represent thermo-fluid dynamic processes in a more accurate way, allowing a very precise representation of scavenging and combustion processes, but the expense of a greater computational burden.

- In cylinder energy balance:

$$\frac{dU}{dt} = \frac{dQ_B}{dt} - \frac{dQ_w}{dt} - p\frac{dV}{dt}$$

- Combustion process (Weibe):

$$\frac{dQ_B}{dt} = H_u g_f \eta_c \left(1 - \exp\left[-a\left(\frac{t - t_0}{t_d}\right)^{b_0+1}\right]\right)$$

- Heat transfer through the cylinder walls:

$$\frac{dQ_w}{dt} = h\left[\pi D(S_c + x) + 2\pi\left(\frac{D}{2}\right)^2\right](T - T_w)$$

- In cylinder pressure:

$$\frac{dp}{dt} = \frac{1}{V}\left[(\gamma - 1)\left(\frac{dQ_B}{dt} - \frac{dQ_w}{dt} - mT\frac{dc_v}{dt}\right) + mT\frac{dR_g}{dt} - p\frac{dV}{dt}\right]$$

Free-Piston Linear Generator – Rebound device model

Rebound device is usually a **gas spring**.
The gas spring model can be obtained as a special as a special case of cylinder model, by excluding:

- Scavenging: rebound device is a closed system
- Combustion processes: no thermal power injected and no changes of the thermodynamical properties (c_v, R_g, etc.) of the gas within the rebound device.

- In cylinder energy balance:

$$\frac{dU}{dt} = \cancel{\frac{dQ_B}{dt}} - \frac{dQ_w}{dt} - p\frac{dV}{dt}$$

- Combustion process (Weibe):

$$\cancel{\frac{dQ_B}{dt} = H_u g_f \eta_c \left(1 - \exp\left[-a\left(\frac{t - t_0}{t_d}\right)^{b_0+1}\right]\right)}$$

- Heat transfer through the cylinder walls:

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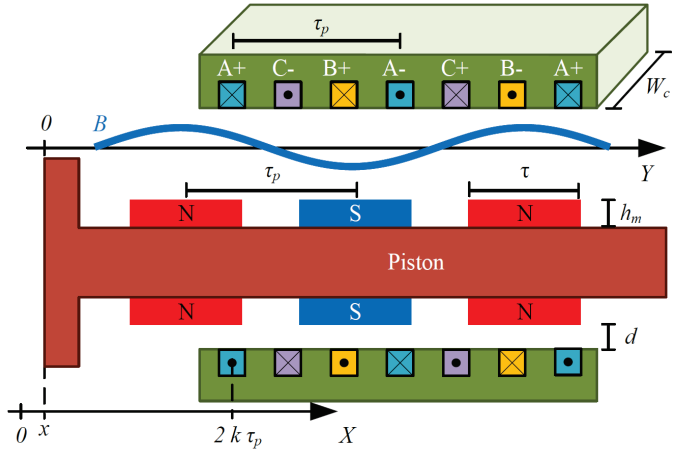
Free-Piston Linear Generator – Linear Electric Generator model

The modeling of the linear electric generator is very similar to the modeling of a rotating electric machine, and is based on three fundamental steps:

- Calculation of the **flux linkages** as a function of the translator position
- Calculation of the **induced voltage** across the windings, by applying Faraday's law.
- Calculation of the **electromagnetic force** acting on the translator, mainly depends on the winding **#currents**.

Flux linkage (phase *a*)

$$N_t \cdot \phi_a = -2 \cdot W_c \cdot B_m \cdot \frac{\tau_p}{\pi} \cdot \cos\left(\frac{\pi x}{\tau_p}\right)$$



Induced voltage (phase *a*)

$$u_a = -\frac{d(N_t \phi_a)}{dt} = 2 \cdot W_c \cdot N_t \cdot B_m \cdot \sin\left(\frac{\pi}{\tau_p} x\right) \frac{dx}{dt}$$

Electromagnetic force acting on the translator shaft

$$F_{lem} = 2HN_tB_m \left[i_a \sin\left(\frac{\pi}{\tau_p} x\right) + i_b \sin\left(\frac{\pi}{\tau_p} x - \frac{2}{3}\pi\right) + i_c \sin\left(\frac{\pi}{\tau_p} x + \frac{2}{3}\pi\right) \right]$$

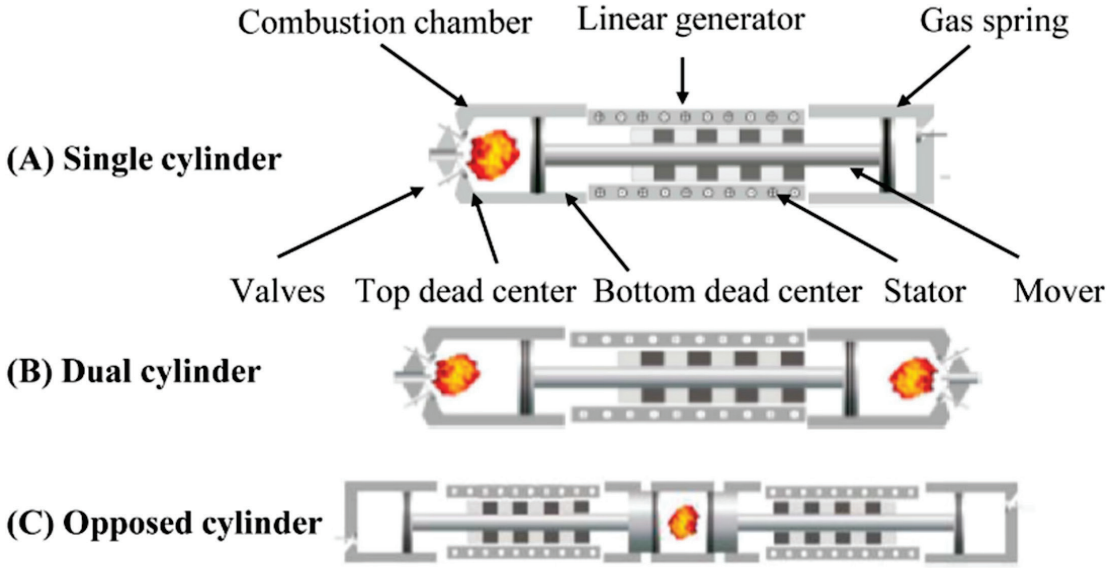
Free-Piston Linear Generator – FPLG architectures

The modeling approach based on the interaction between subsystems allows to easy represent the operation of different FPLG architectures.

Single piston:
- 1 combustion chamber;
- 1 linear electric generator;
- 1 rebound device.

Dual piston:
- 2 combustion chambers;
- 1 linear electric generator.

Opposed piston:
- 1 combustion chambers;
- 2 linear electric generator;
- 2 rebound devices.



If the model of each subsystem corresponded to a LEGO® brick, the model of any FPLG would be like a LEGO® building.

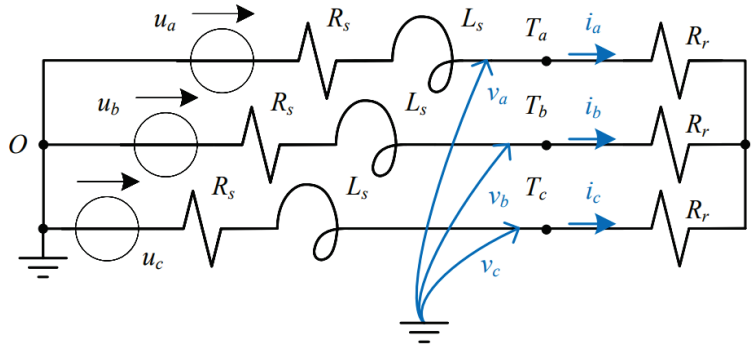


Free-Piston Linear Generator – Electrical load

The keywords **#convert** and **#current** suggest that a fundamental element is missing in the previously set modeling criteria: the **#electrical load**, or better the model of the electrical system connected to the terminals of the linear electrical machine.

- **#convert**: the electrical subsystem is necessary to complete the energy balance.
- **#current**: they are necessary for electromechanical conversion to take place.

- It is necessary to connect the linear electrical machine to a proper load.
- The usual approach is to represent the electrical load with an equivalent set of **three-phase Y-connected resistors**.



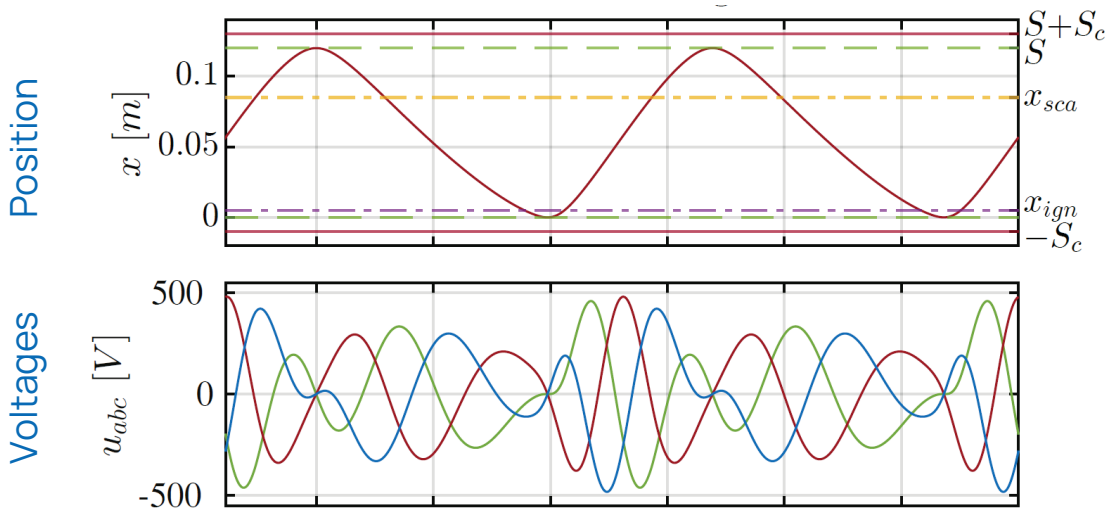
Electric load and electric power conversion

03

Free-Piston Linear Generator – Electrical load

The FPLG electrical output is an **#AC #unregaluted source**:

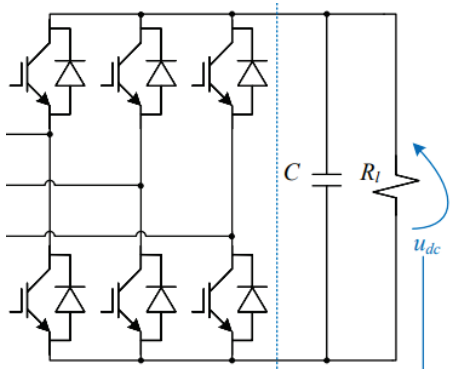
- variable **frequency**: it depends on the FPLG frequency.
- variable **amplitude**: it depends on the FPLG speed.
- variable **waveform**: it depends on the FPLG motion.



In real applications, it is necessary to supply the load with a regulated AC or DC voltage:

- hybrid vehicles require a constant voltage DC bus;
- on grid applications require AC three-phase balanced voltages, with constant amplitude and constant frequency.

One or more conversion stages are necessary to properly supply the load.



Are we sure that an active rectifier operates like a resistive load?

Free-Piston Linear Generator – Electrical load

Resistive load

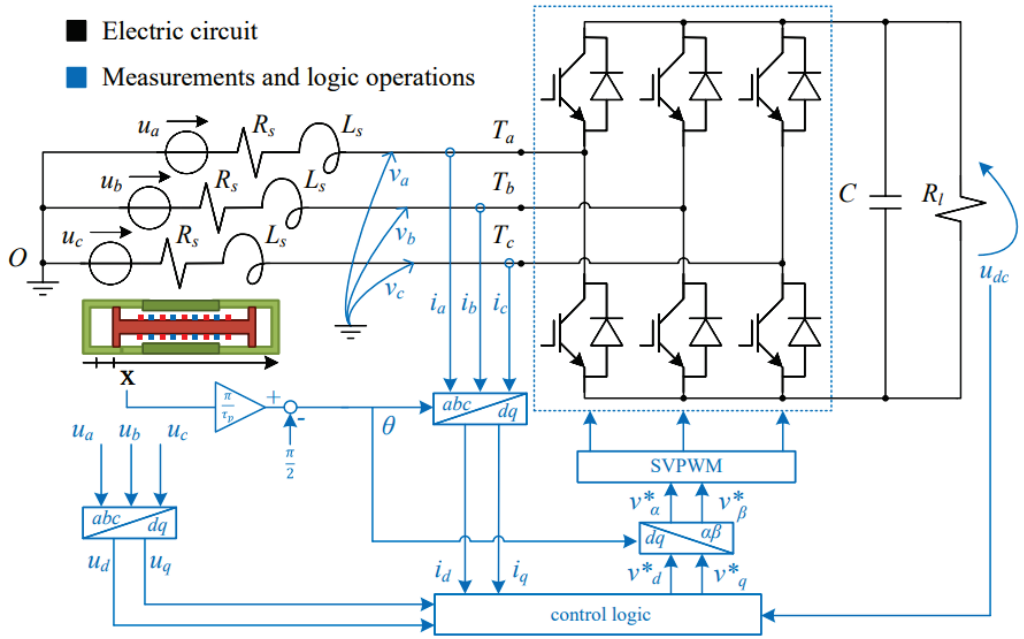
- Electromagnetic force is proportional to the current.
- Current is proportional to the induced, or no-load, voltage (negligible reactance).
- Induced, or no-load, voltage is proportional to the speed (Faraday's law).
- As a consequence: **electromagnetic force is proportional to the speed.**

The effect of a balanced resistive load is almost identical to viscous friction.

Viscous friction, thus balanced resistive load , tends to stabilize the FPLG.

Active rectifier

- Electromagnetic force is proportional to the current.
- The current depends on the power converter control.



Free Piston energy conversion systems modeling

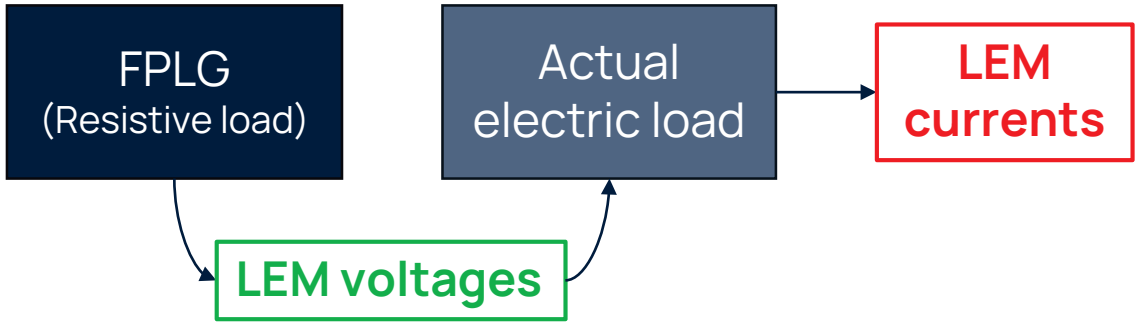
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Free-Piston Linear Generator – Electrical load

Usual modeling approach

It is based on the hypothesis that the electromagnetic force is independent on the electrical load, and it is equal to the electromagnetic force obtained in the case of equivalent resistive load.

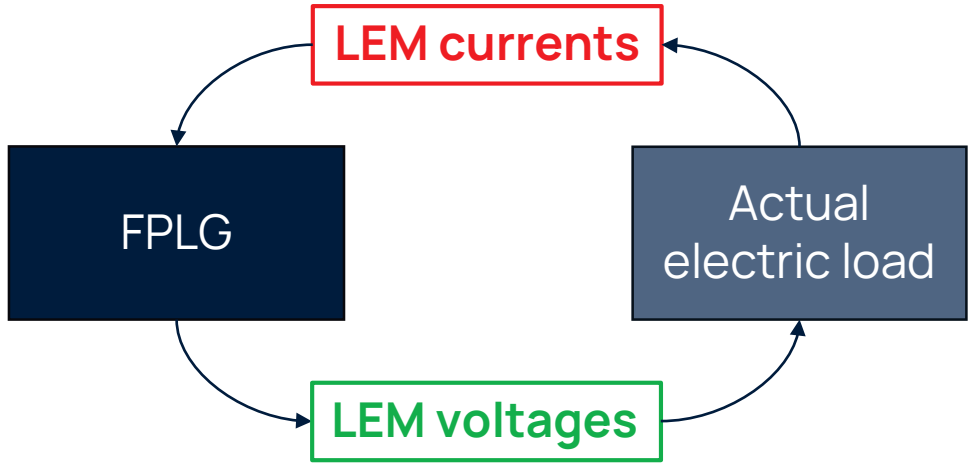
- FLPG model allows the no-load voltages calculation;
- No-load voltages are applied to the electrical circuit representing the actual load, actual LEM currents are calculated.



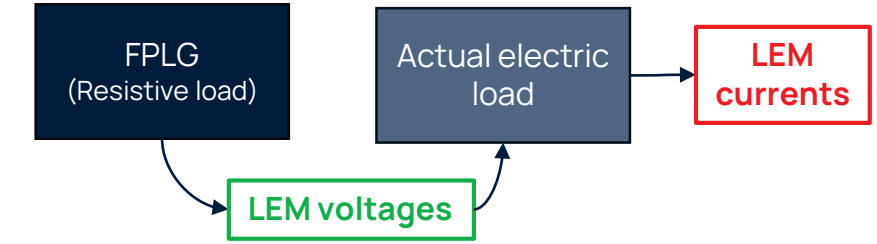
Full modeling approach

FPLG model and actual electric load model are integrated in a sort of closed loop.

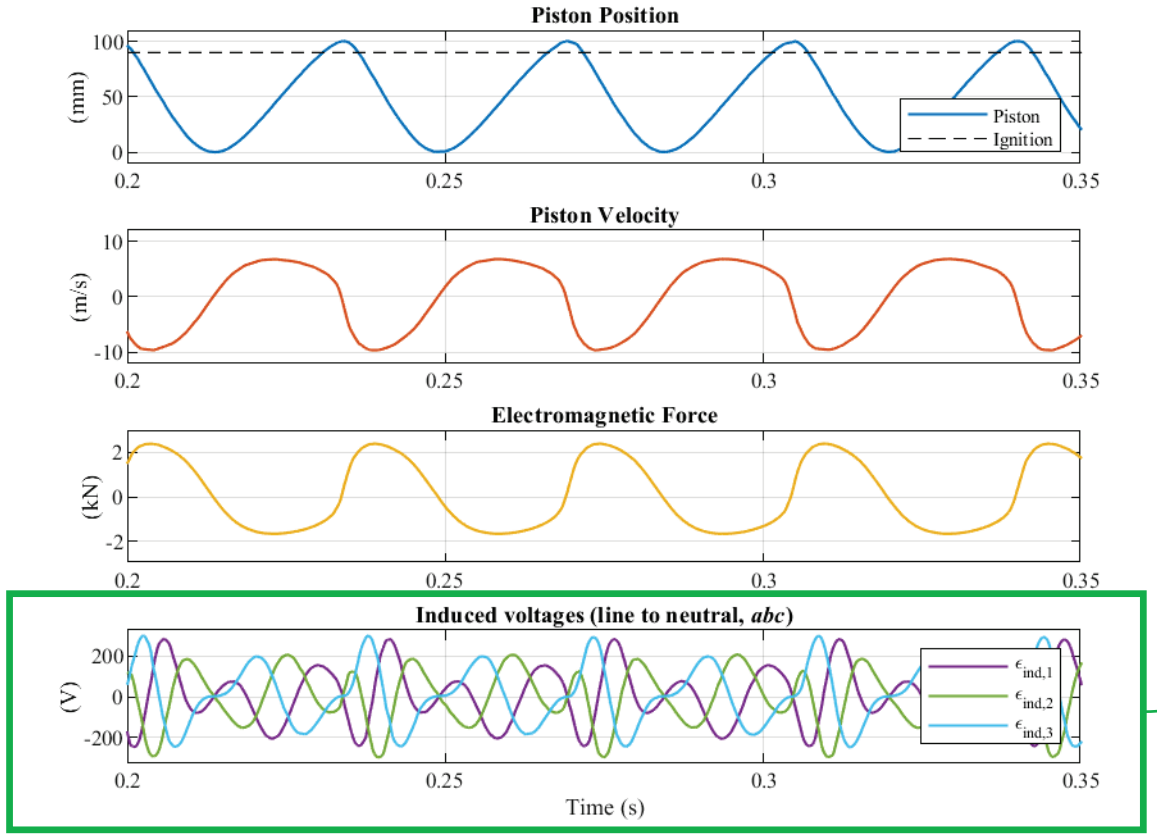
This approach allow the precise evaluation of the electromagnetic force due to the current absorbed by the specific load connected to the LEM.



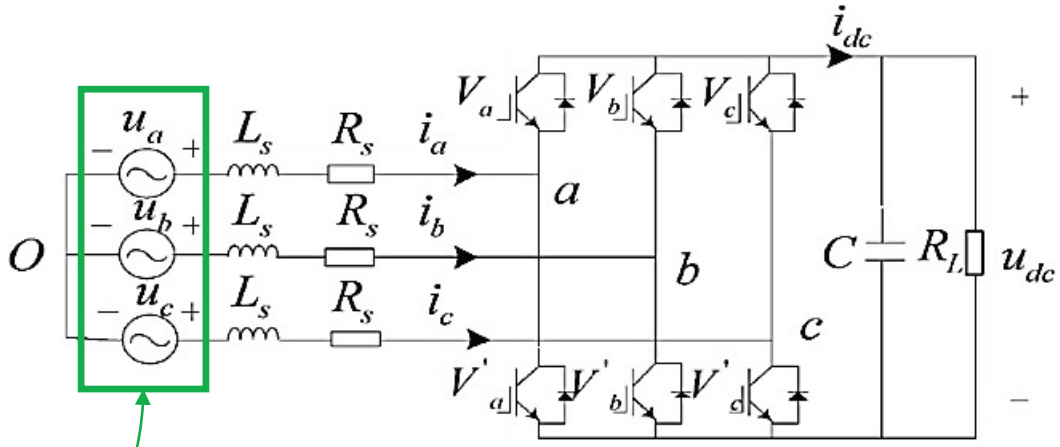
Example 1a
Usual modeling approach and active rectifier



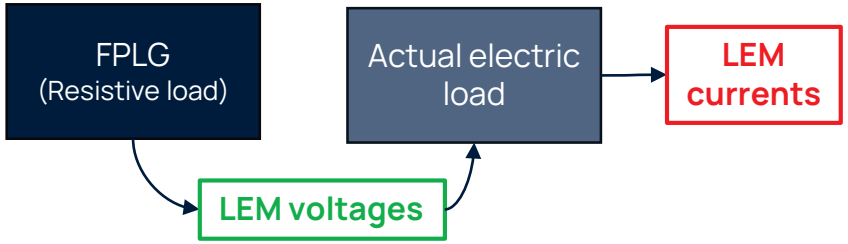
Position, force and no-load voltage results



Electrical load

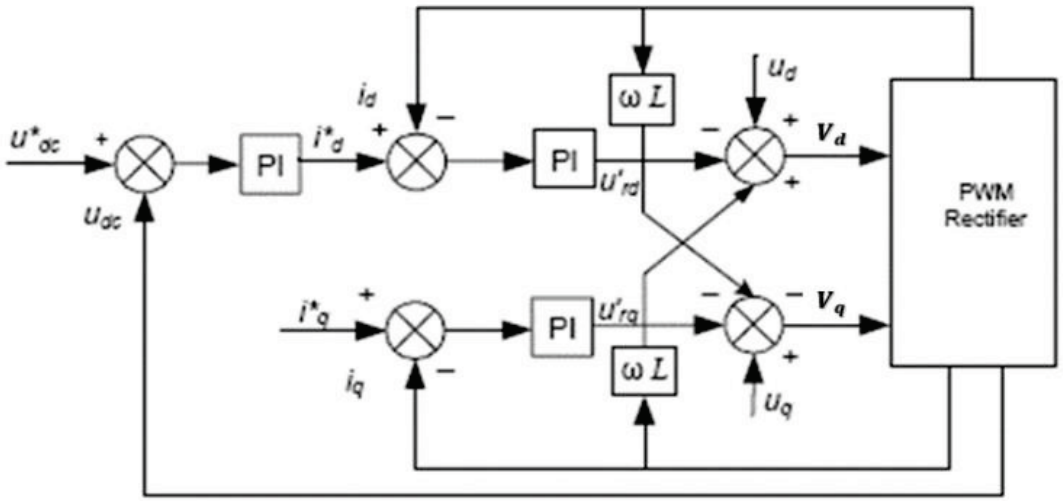


Example 1a
Usual modeling approach and active rectifier



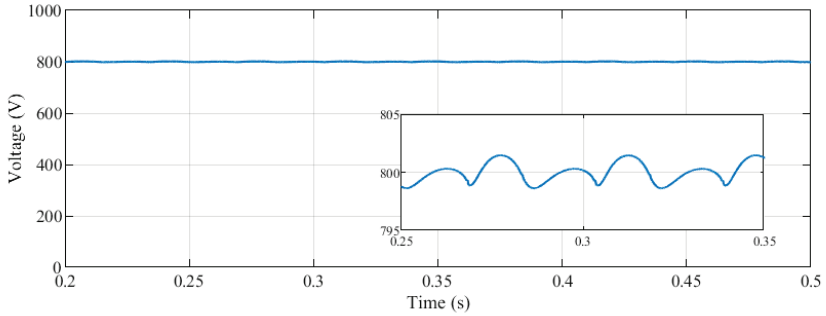
Active rectifier control scheme

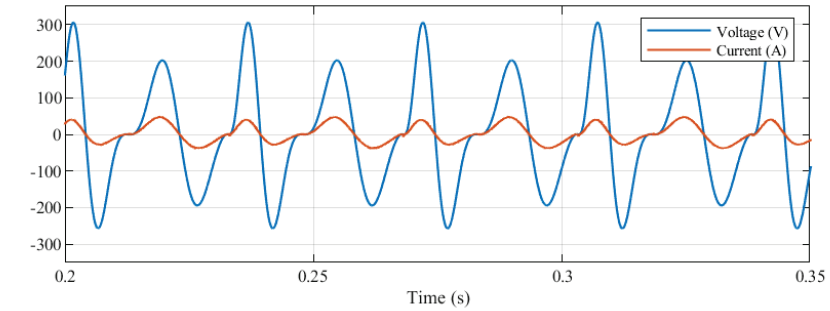
- DC side voltage has to be kept to the reference value U^*_{DC} = 800 V.
- The in-quadrature component reference current, is i^*_q = 0 A.



Results

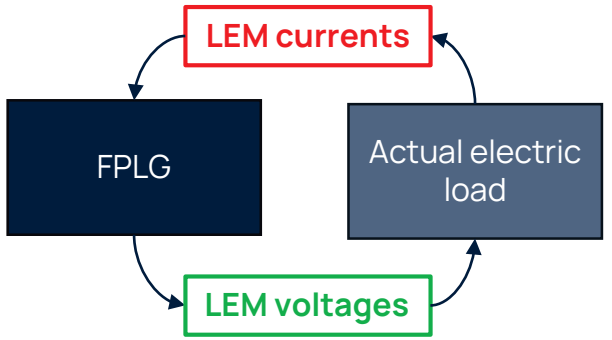
DC voltage is stable. *Results seem to be good…*



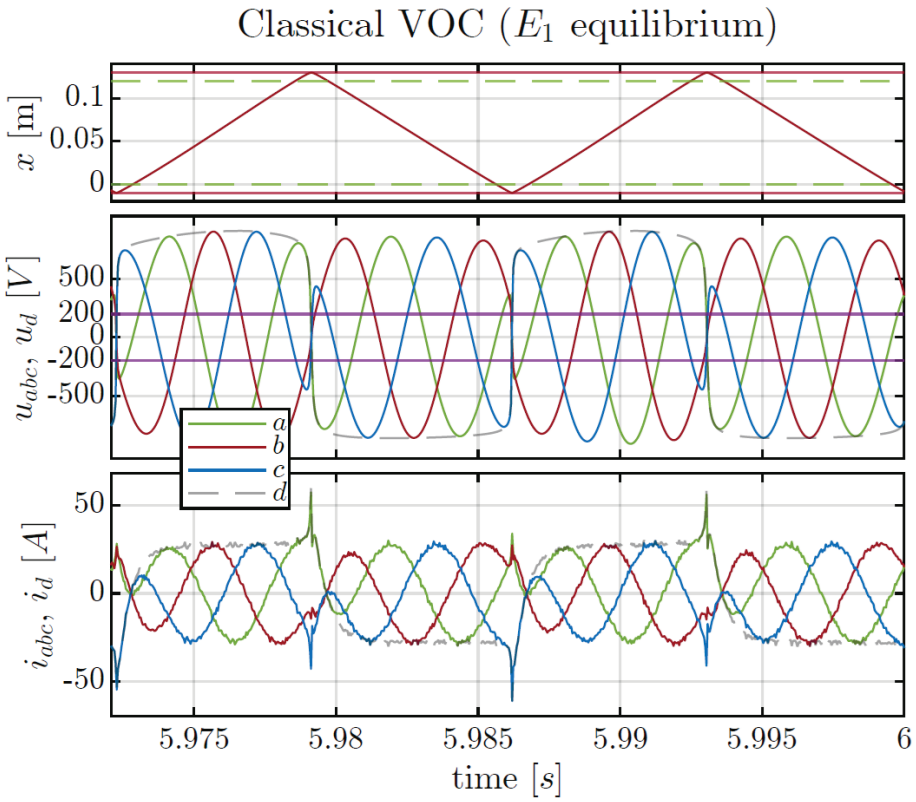


… but something is strange. The higher the current peak, the lower the voltage peak, opposite than expected for a resistive load.

Example 1b
Integrated modeling approach – Active rectifier



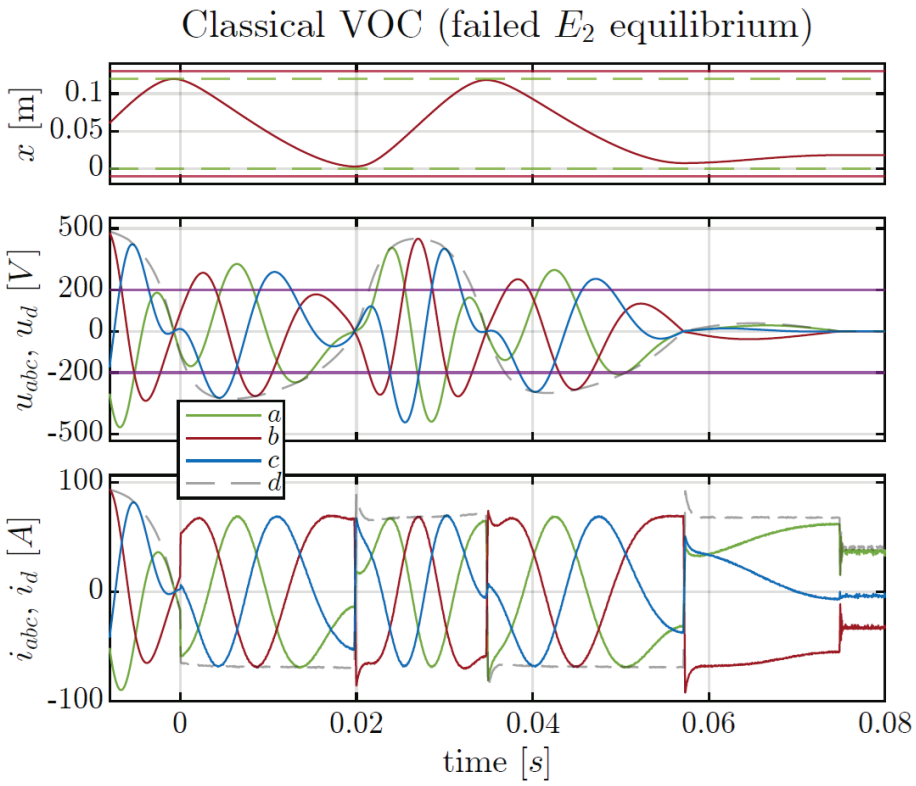
Stable operation (controller setting A)



High current near the top and the bottom dead center results in a large electromagnetic force when the piston velocity has to invert.

In case of resistive load, the electromagnetic force is zero at both the the top and the bottom dead center.

Unstable operation (controller setting B)



Free-Piston Linear Generator – Control

The FLPG is a complex system, and the stability of its cycle is not robust by nature.
Control techniques are necessary to maintain FPLG operation and to optimize the energy conversion process.

An **#integrated model** involving:
- Free Piston Linear Generator;
- Actual electric load (power converters, etc…);
- Control system.

is fundamental for the development of effective control techniques.

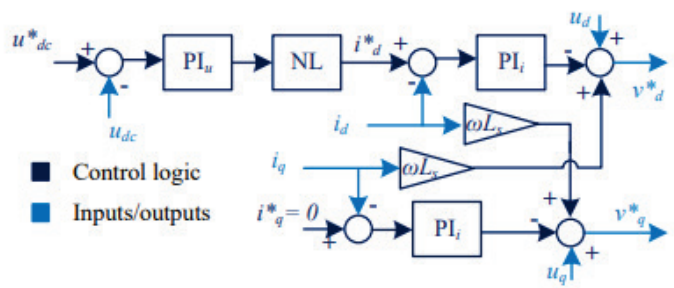
Control can be actuated on:
- The ICE (e.g. modifying the time of the ignition)
- The rebound device (e.g. increasing or reducing the internal pressure)
- The electrical load (e.g. varying the current in the LEM through power converters).

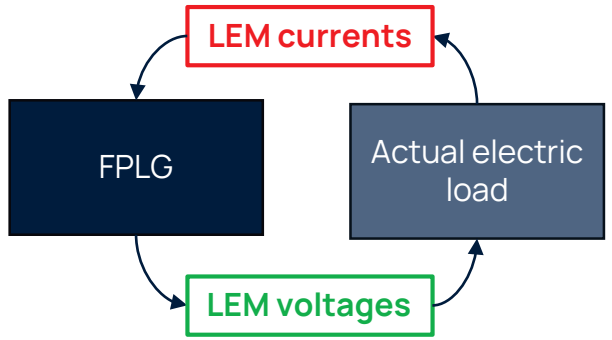
Concerning the control applied to the electrical circuit:
- The use of fully controlled power converters, such active rectifiers, as the first conversion stage requires the development of reliable control strategies.

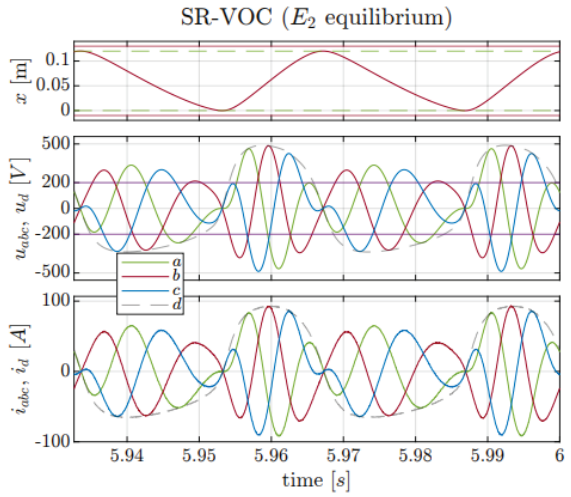
Example 2
Integrated modeling approach

Model simulations, control design and data analyses led to the development of a more robust, still simple control technique, also reducing the commissioning phase.

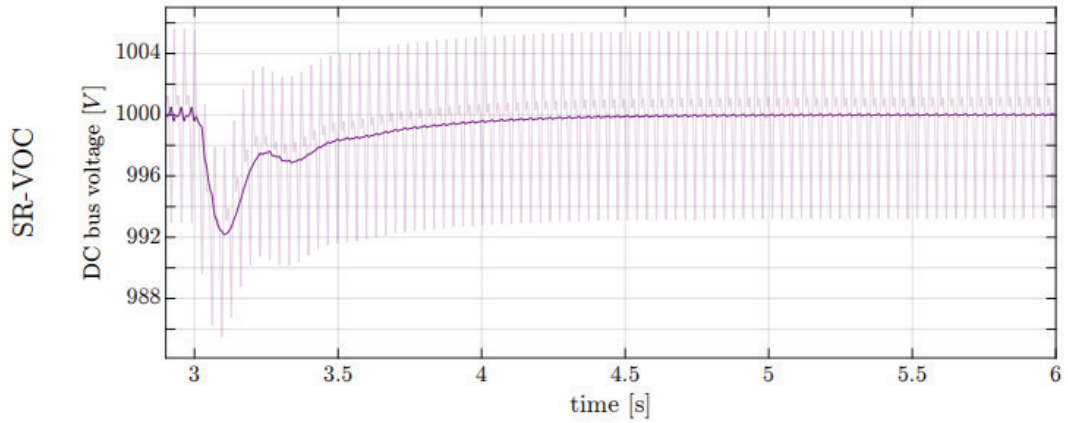
Since the effect, in terms of electromagnetic force, of a balanced resistive load is almost identical to viscous friction that stabilize the FPLG, it is possible to test an active rectifier control scheme which mimics the behavior of a three-phase resistor.







- Current and voltage are in-phase.
- Operation is stable.
- Large DC voltage ripple.



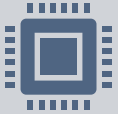
Conclusions, future works, and challenges

05

Conclusions



Integrated modelling and multidisciplinary approach is fundamental to study and simulate complex systems.



The analysis and the design of Free Piston Linear Generators requires a **#multidisciplinary** and **#holistic** approach to simulate the actual behaviour of the system in a real-word scenario, and to implement and test robust controls.

Free Piston Linear Generators is more than the sum of its subsystems.



Future research works will focus on the more precise simulation and the optimization of Free Piston system components, as well as the study of more complex configurations.

The goal is to develop an effective technology, suitable for the market.

Challenges and still open questions

Electric load and power converter control

The resistive load - real or emulated through the power converter – results in electricity generation both during the expansion and the compression strokes. Is it really worthwhile to use some of the energy stored in the rebound device to generate electricity?

Linear Electric machine control

The operation of the linear electric machine is similar to that of a rotating machine that continues to oscillate around a specific position, like a pendulum.
Is it really appropriate to use FPLG control strategies initially designed for the control of the angular velocity of rotating machines, or for the networking of generators?

Linear Electric machine design

Is the sinusoidal magnetomotive force distribution the best solution for the linear electrical machine? Are there any MMF distributions that can result in more suitable induced voltage waveforms?

FPLG configurations

What are the advantages and disadvantages of different FPLG architectures?
Are there any FPLG configurations that increase system stability?



The Linear Electric Machine is not just the linearized version of a rotating machine.

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